Heterogeneity Measures and Secondary Delays on a Simulated Double-Track

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Abstract
The demand for transportation on railways grows for each year and many railway lines are already used close to maximum capacity. One way to increase capacity is to reduce traffic heterogeneity. Heterogeneity is introduced when train services with different speeds operate on the same line. There are many definitions of heterogeneity in literature. Good measures are important in order to be able to quantify capacity lost due to heterogeneity, and consequently how capacity can be gained by reducing it. This paper analyse some of the existing measures as well as introduces a new one, Mean Pass Coefficient (MPC). Other measures analysed are: number of speed levels (SL), speed ratio of fastest to slowest train (SR), mean difference in free running time (MDFR) as well as sum of shortest headway reciprocals (SSHR) and sum of arrival headway reciprocals (SAHR).

Two infrastructure models of double-track lines with overtaking stations spaced at different intervals are simulated. A large number of timetables are created where traffic density as well as the mix of slower and faster trains is varied. Each timetable is characterized using the different definitions of heterogeneity and the results are used in regression analyses to determine their explanatory value with respect to secondary delays created in the simulations. Results show that MPC performs best closely followed by MDFR and SR, while SL is worse when it comes to explaining secondary delays. SSHR and SAHR also show good performance. The performance of the measures increases when primary delays are high, but is unaffected by interstation distance.

Keywords
Railway, delay, heterogeneity, simulation, regression

1 Introduction

The ever increasing demand for transportation on railways makes it important to understand how railway operation reacts to increased capacity utilisation. One factor that is of great importance when capacity is discussed is traffic heterogeneity. Heterogeneity can be used to describe two different properties of the timetable. The first is how evenly distributed the train movements are over a given period of time and the second is associated with speed differences between trains. In heterogeneous timetables, trains use the infrastructure unevenly over time with great difference in average speed. Besides limiting the number of trains it is possible to schedule, high heterogeneity does also increases the risk for delay transfer, i.e. secondary delays. In the first case, the buffer times between trains are unnecessary small. In the second the speed difference implies that faster trains risk catching up on slower trains and that slower trains may be forced to stand aside for unscheduled overtakes.

When a double-track railway line with heterogeneous traffic has reached its maximum capacity, one option is to increase capacity by reducing heterogeneity. One way to this is to reduce the mean speed of the fastest trains or increase it for the slowest. Other options are to separate slow and fast trains in time or space, e.g. by letting slow freight trains run at night and faster passenger trains during the day or by constructing new tracks. Each solution costs and it is therefore important to be able to quantify how heterogeneity affects capacity. This in turn requires that it can be measured in good way. Many definitions of heterogeneity can be found in literature and the main objective of this paper is to analyse some of these and how well they can be used to explain secondary delays in double-track operation.

There are several different methods for railway operation analysis. They involve simulation, optimisation, queue theory and other analytical methods as well as statistical analysis of empirical data. All methods have their specific strengths and weaknesses and use models with different levels of detail. In general, methods based on less detailed models may be better for drawing general conclusions, which make them suitable tools for long term planning. On the other hand, more detailed models are required to perform thorough studies, but they do also require more data as input and risk generating results that are only valid for a specific setup.

In this paper micro simulation is used in an extensive experiment where several parameters are varied. Simulation results are used for regression analysis to determine the performance of several heterogeneity measures under different conditions. The following sections cover related research, description of the simulation experiment and definition of heterogeneity measures. In the section covering the results, some illustrative examples from the simulations are given to increase the understanding before the results of the regressions are presented. Finally some general conclusions are made.

2 Related Research

Huisman [5] developed a stochastic model for estimating the running time on double track railway lines with heterogeneous train traffic. The model describes secondary delays due to faster trains catching up with slower ones. The train order can be either random, which is useful for long term planning, or defined by a cyclic timetable. The primary delays used include both entry delays and running time extensions. Huisman demonstrates the model by applying it on a Dutch railway line to show how the number of trains, heterogeneity, primary delay, train order and buffer times influence the delays. However, the model is limited to analyse delays on line sections where trains are not allowed to overtake, hence delays at stations due to overtaking and dispatching actions are not included.

Gorman [3] uses real data to do statistical estimations of delays. He predicts total train running time based on free running time predictors and congestion-related factors, such as meets, passes, overtakes, train spacing variability and departure headway. He concludes that the factors showing largest effect on congestion delay are meets, passes and overtakes.

Gibson et al [2] develops a regression model using delay data from the rail network in the UK. He uses a method similar to the timetable compression defined in the UIC 406 leaflet [19] to define capacity utilisation. He tests a number of functional forms (exponential, adjusted exponential, power and linear), and finds that secondary delays increase exponentially with capacity consumption on a line section. He also discusses how the relative speed of a train affects its marginal cost, congestion cost. Using simulation, he concludes that adding a train that is 20 % faster than the fastest train in the timetable, have a congestion cost that is 20 % higher than predicted for a train of average speed. Similarly, adding a train that is 20 % slower, costs 50 % more than the average train.

Vromans [21] defines two measures of heterogeneity and uses simulation to show their correlation to the average delay. The two measures are SSHR (sum of shortest headway reciprocals) and SAHR (sum of arrival headway reciprocals). The first measure looks at the headway both at the start and at the end of the line section, and therefore takes into consideration both the heterogeneity in speed of the trains and the spread of the trains over time. The second measure, SAHR, focus only at the headway at the end of the line section under the assumption that the headway at the end is more important than at the start. Several timetables with different heterogeneity are created and simulated using the simulation tool SIMONE to show that both heterogeneity measures correlate positively to the average delay. In the simulation both dwell time extensions and running time extensions are used. Overtakes are also possible. In [20], Vromans further develop the measures by compensating for the minimal headway that is technically possible between two trains at the each location, thereby estimating the headway buffers rather than the absolute size of the headways. This has an advantage if the mimum technical headway varies along the line or between different train types. The SSHR and SAHR are further developed by Landex [7] into new measures for heterogeneity that is independent of traffic density and number of trains used in the calculation.

Murali et al [13] develops at simulation-based technique to generate delays used in regression models to predict delays in double- and single-track sub networks. Several parameters are used to describe the topology of the network as well as the operating conditions when the train of interest enters the subnetwork. They find an exponential relationship between delay, train mix and parameters describing the operating conditions and network topology.

Lindfeldt [12] uses advanced experimental design, simulation and response surface metamodelling to analyse how nine different parameters affect delay development of mixed traffic on a double track railway line. The investigated parameters are: distance between adjacent overtaking stations, train top speed, train frequency, entry delays and running time extensions, for both high-speed services and freight services independently. In order to reduce the number of necessary parameters, the delays are modelled by negative exponential distributions. In addition, Lindfeldt points out the difficulty of defining the timetable by a few independent factors. Thanks to the experimental design using Latin hypercubes, only 66 design points are needed to form the metamodels. The simulations are performed in the simulation tool RailSys using the mean and standard deviation of the delays as response variables. The results show that the speed and frequency factors as well as the running time extension have great impact on delays. The entry delays and inter-station distance are found to have less impact.

Sogin et al. [18] analyse the effect of heterogeneous traffic on a single track freight network. The analysis is performed with a micro simulation software called Rail Traffic Controller, RTC, and the measure of performance is delay of the freight trains in min per 100 train miles. The delay includes both times for meets and passes, i.e. they are not planned in advance, and are calculated by RTC. Traffic density is varied and heterogeneity is controlled by systematically adding passenger trains of different speeds. For completely homogenous freight traffic, delays are found to increase exponentially with traffic density. A relationship between speed difference between trains and delays of the slower trains is proposed. At higher traffic densities, the delays of freight trains increase with speed difference, but with high enough speed difference, the effect diminishes.

Yung-Cheng et al [22] creates parametric models to estimate capacity of single and double-track operation. RTC is used to perform a full factorial design. Traffic consists of freight trains that are operated without a timetable and stochastic entry delays are applied in the simulations. Output from the simulation, train delays, is used to estimate parameters in both regression models and in a neural network (NN) model. Factors in the model for single track operation are siding spacing, signal spacing, track speed, volume (trains/day) and heterogeneity. For double-track, the factors are crossover spacing, signal spacing track speed, volume, and heterogeneity. A measure of heterogeneity is defined that is applicable if the traffic mix consists of two types of trains. The conclusion is that the regression model performs better in estimating single track operation while NN is better on double-track operation.

Even if there are many papers including heterogeneity in the analysis, there are few that discuss alternate ways of measuring it. Much work has been done to create parametric models that explain secondary delay, but many of them are complex with many factors and the effect of heterogeneity and the performance of the used measures are not always easy to isolate. It is common that studies of freight train operation do not model the timetable in such detail as is needed if the results should be applicable to passenger traffic, where it is important to separate scheduled delay from operational delay and model the effect of timetable allowance. In this work all trains are operated according to conflict free timetables, hence the effect of heterogeneity can be separated into scheduled delay and operational delay.

3 Methodology

In this work the railway simulation tool RailSys [15] is used to perform the simulations. It is a tool for microscopic simulation and timetable planning and it is shown in previous work that it is capable of generating realistic results when calibrated and used to simulate real operation [11, 17]. When microscopic simulation is used, it is common that the analysis consists of comparing results from a few simulated scenarios where properties of the infrastructure, timetable or perturbations are varied. Each scenario requires a new simulation and it can be very time consuming if the number of scenarios is too high. If the aim of the analysis is to make general conclusions not connected to a specific timetable, simulating many timetables helps making the results less timetable dependent.

To handle experiments with many scenarios, an interface is required to handle input and output from the simulation tool RailSys. A method to transfer timetable and perturbation data into RailSys using xml files is developed in [9], and is utilised in this paper to handle the hundreds of scenarios of the factorial experiment. Secondary delays and used allowance is estimated from actual delays generated by the simulations.

3.1 Experimental Setup

A factorial experiment is performed with a large number of timetables, two different infrastructure variants, and two levels of primary delays, table 1. The infrastructure models consist of one track operated in one direction, thus mimicking the operation of a double track with assumed independency of traffic in different directions. Overtaking stations are spaced equidistantly. The timetables are defined as cyclic timetables of up to three trains per cycle using up to three different train types, high speed, intercity and freight trains. Taking cyclicity into account, this makes in total 14 unique combinations, i.e. types of cyclic timetables with different mixes of train types and therefore different degrees of heterogeneity. In the scheduling algorithm the timetables are controlled by the starting order of the trains and their headway at the origin. The perturbations include three different types of delays, entry delay, running time extension and dwell time extension. All three types are varied coherently for two levels and are based on distributions from a previous project using empirical data from real operation in Sweden [14]. In the experiment, explanatory variables are inter-station distance, heterogeneity, number of trains per hour and level of primary delays. Dependent variables are scheduled delay, secondary delay and used allowance. The scheduled delay is a property of the timetable and it is consequently not necessary to perform a simulation to obtain it. The allowance consists of two parts, running time allowance and allowance at stations where trains are scheduled to stop. It is especially the allowance at stations that is dependent on traffic density and heterogeneity of the timetable, due to the frequency of scheduled overtakes.

Table 1: Experimental setup. A full factorial design with 336 scenarios. The first factor is the distance between overtaking stations. The second is type of timetable, i.e. which train types are included in the timetable (heterogeneity factor). Third factor is traffic density which is used to vary headways between the trains. Fourth, the amount of primary delays applied in the simulation.

Inter-station distance		**Train-type start order**	**Traffic density**	**Perturbation level**
[km]	Timetable number	1: high-speed, 2: intercity, 3: freight	Headway, % of minimum	Low, high
	1	1		
	2	2	100	
	3	3		
20	4	1 2	116	
	5	1 3		Low
	6	2 3	138	
	7	1 1 2		
	8	1 1 3	171	
	9	1 2 2		
40	10	1 2 3	223	High
	11	1 3 2		
	12	1 3 3	322	
	13	2 2 3		
	14	2 3 3		

Secondary delays and how allowances in the timetable are used are dependent of the timetable and applied primary delays. Since the primary delays are modelled by a stochastic process, simulation is needed to obtain them, figure 1. The timetable is characterised by no. train/h and the different measures for heterogeneity. The minimum headway referred to in table 1 is dependent on type of timetable, i.e. train order, and the inter-station distance. After the timetable is generated, the scheduled delay is evaluated. The available allowance is the sum of running time allowance, allowance at stations and scheduled delay.

Results from the simulation are obviously dependent on many parameters. The most significant may be the scheduling scheme used to create timetables, dispatching rules in the simulation model, how primary delays are modelled and infrastructure layout. For example, different schemes when timetables are created may distribute allowances and buffer times differently. Other dispatching priorities in the simulation affect how trains are operated in the simulation and infrastructure parameters such as signal block lengths and number of tracks at stations affects capacity. All this factors influence secondary delays and is important to keep in mind when results are analysed.

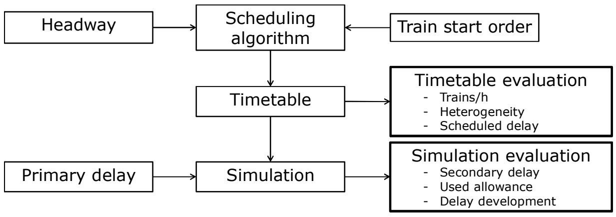

Figure 1: Workflow of the experiment.

Infrastructure Model

The models are simplified double-track lines with only one track since the traffic is only simulated in one direction. This simplification is rational since in Sweden traffic in different directions is generally independent of each other. The stations are modelled as two-track overtaking stations with a track length of 1000 m. The total length of the line is constant at 200 km, hence does the number of stations vary between the infrastructure variants. All tracks are completely horizontal. The lengths of signal block sections are 1000 m and no overlaps are required for releasing train routes. The speed is 200 km/h on the main track and 100 km/h on the sidetrack.

Timetable

As mentioned before, the timetable is made up of three different types of trains: high-speed, intercity, and freight trains. Some characteristics of the trains are listed in table 2. The total number of stops is independent of infrastructure variant and all trains stops at the first and last station. Both when timetables are created and in the simulation, faster trains have higher priority.

Table 2: Train type characteristics.

Train type	High-speed	Intercity	Freight
Vehicle	X50	X60	RC4, 1000 ton
Top speed [km/h]	200	160	100
Average speed [km/h]	168	125	95
Total running time [min]	72	90	127
Number of stops	4	6	2
Priority (1:high, 3:low)	1	2	3

All scheduled stops are modelled in the same way for all train types. The scheduled dwell time is 120 s and the minimum dwell time is 30 s. The difference between scheduled and minimum dwell time is an allowance used to compensate for delays. On line-sections, running time allowance is applied by increasing the scheduled running time. A running time allowance of 6% is added to the minimum technical running time, the same as used by DB [16]. The trains are allowed to use the entire allowance to catch up delays. The cyclic timetables consist of 35 cycles, where the five first and last cycles are considered to be warm-up and cool-down periods and discarded in the evaluation.

Delay Modelling

The primary delays applied in the simulations are entry delays, running time extensions and dwell time extensions. Both entry delays and running time extensions are modelled by empirical distributions and dwell time extensions follow an analytical lognormal distribution. All distributions is taken from an earlier project where empirical data from the Western Main Line in Sweden (a double track line from Stockholm to Gothenburg) is used to for estimations [14]. The distributions have in some cases been adjusted according to the new infrastructure they are being applied to. In some cases they have also been altered in order to reduce the number of replications needed in the simulation to achieve stability. The same delay distributions have been applied to all train types. In simulations of real train operation, different train types are often allocated to different distributions due to different behaviour in real life. This is especially true when comparing freight trains and passenger trains. In this work however, the focus is on how parameters such as traffic density and heterogeneity affect different types of trains. Therefore, in order to keep the results comparable between train types, the same primary delays are applied to all trains. This also applies to how stops are modelled.

Table 3: Primary delay distributions used in the simulation.

Perturbation level	Location	Distribution type	Mean [s]	Std [s]	alpha	beta
Low	Entry	Empirical	120	180	-	-
	Line (20 km)	Empirical	5.1	23	-	-
	Line (40 km)	Empirical	10	32	-	-
	Station	Lognormal	-	-	3.92	0.56
High	Entry	Empirical	240	390	-	-
	Line (20 km)	Empirical	9	30	-	-
	Line (40 km)	Empirical	18	41	-	-
	Station	Lognormal	-	-	4.29	0.56

In the simulation model, running time extensions are applied between all stations (line sections). It is reasonable to assume that the magnitude of the delays is proportionate to the length of the line sections [4]. The input distributions are based on empirical data from line sections with an average length of 34 km and have been rescaled to fit the length of the line sections in the model (20, and 40 km) by adjusting the probability of receiving a delay. The scaling is done in such a way that the mean value of the sum of all running time extensions applied along the whole line is the same, i.e. the mean of the applied running time extensions are independent of infrastructure variant. The same is not true for the standard deviation and is an effect of rescaling the distributions. When a dwell time extension is applied, the value given by the stochastic process is added to the minimum dwell time to obtain the minimum time that the train has to stop. The distributions are adapted to stops modelled by a minimum dwell time of 30 s and scheduled dwell time of 120 s. In reality however, a minimum dwell time of 30 s may be too short for long freight trains where releasing the breaks after applying them for a complete stop takes a considerable amount of time. However, as previously discussed, this is not considered and stops are modelled in the same way for all train types to simplify the analysis of the simulation results. Table 3 shows some properties for the different distributions. The simulation is run for 80 replications to achieve stability.

3.2 Heterogeneity Measures

Speed Levels (SL)

As the number of trains with different speeds increase, so does the complexity of the timetable. A complex timetable is complex to operate and demand more complex dispatching resolutions and might generate more secondary delays. For this reason, the first heterogeneity measure is number of speed levels present in the timetable. It does not consider how much the speeds differ, just the number of unique speeds. If it is used over a longer line, average speeds can be used when determining the SL to capture the effect off different stopping patterns etc. SL is always equal to or larger than 1 (1: homogenous timetable).

Speed Ratio (SR)

The Speed Ratio (SR) is proposed and used by Krueger in a parametric model for estimating delays [6]. It measures heterogeneity in speed and is the ratio of fastest to slowest train speed, eq. below. It is a measure that does not rely on cyclicity and can easily be applied to real timetables. A similar measure is proposed in [8] where the speed ratio is calculated as the 0.95 percentile divided by the 0.10 percentile, thus avoiding the most extreme speeds that may be present in a real time table. I this paper timetables include at most three different train types, hence there are never more than three different speed levels and the two measures give the same results. A drawback may be that it does not capture the full variety of the train speeds. In a homogenous timetable where all trains travel at the same speed, SR has a value of 1.

$$SR = \frac{\max(v_1, v_2, \ldots, v_n)}{\min(v_1, v_2, \ldots, v_n)} \tag{1}$$

Mean Difference in Free Running Time (MDFR)

The Mean Difference in Free Running time (MDFR) is a measure developed in [9] where it complements traffic density when several different aspects of timetables and delays are discussed. It utilises *free* running times, i.e. running times when trains are not affecting each other in terms of overtakes. Hence, it does only describe heterogeneity in speed and is independent of traffic density. MDFR is given by the eq. below and is calculated as the mean value of the differences in running times between trains in the cycle. The difference in running time indicates the consequences of faster trains catching up on slower trains on line sections and the time a low priority train can be expected to wait to be passed by a faster train. Together with the headway, the difference in running time does also hint on the required number of overtakes. The difference is calculated for all combinations of trains in the cycle, not only between adjacent trains. For this reason the measure can be expected to perform best for timetables with short cycles and when trains suffer from significant delays. When this is the case, it is probable that all trains in the cycle are likely to affect each other.

$$\begin{aligned} MDFR &= \binom{n}{2}^{-1} \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} abs(rt_j - rt_i) && n \geq 2 \\ MDFR &= 0 && n < 2 \end{aligned} \tag{2}$$

In the same paper another measure based on the actual scheduled running time, i.e. including overtakes and therefore not independent of traffic density, is presented: Mean Difference in Scheduled Running time (MDSR). However, the MDFR was found to have higher explanatory value due to that it is independent of traffic density. In a homogenous timetable where all trains travel at the same speed, MDFR has a value of 0. It is not independent of absolute speed or distance.

Mean Pass Coefficient (MPC)

The Mean Pass Coefficient is developed in this paper and measures speed heterogeneity. It is designed to predict the number of overtakes needed in a cyclic timetable, with the assumption that each overtake introduces dependencies between trains that might cause delay transfer. The number of overtakes is achieved by multiplying MPC by the number of trains/h supported by the timetable. It is the mean value of two figures, pass coefficient (*psc*) and passed coefficient (*pdc*), each calculated individually for each train in the cycle, eq. (3). The *psc* indicates the number of times a train pass other trains and the *pdc* the number of times it is passed by other trains.

If the coefficients are based on free average speeds and running times, they become independent of traffic density. However, since increasing traffic densities introduces more overtakes and hence lowers the mean speed of lower priority trains, the error in the estimated number of overtakes grows as traffic density increases. If the aim is to estimate the number of overtakes, the actual running times and mean speeds of the scheduled timetable should be used instead. Then the measure is no longer independent of traffic density. Another assumption that may limit the accuracy of the predicted number of overtakes is that the cycle time of the timetable should be short, compared to the time it takes the trains to run the whole line. The measure is continuous, while the number of overtakes scheduled in the timetable is discrete. This is not necessarily an disadvantage, since the number of overtakes actually realised in the operation may not always be the same as the scheduled, due to delays and dispatching etc. When MPC is referred to in this paper, it is based on *psc* and *pdc* using free running times and speeds. In a homogenous timetable where all trains travel at the same speed, MPC has a value of 0. It is not independent of absolute speed level or distance.

$$
\begin{aligned}
psc_i &= \frac{1}{n}\sum_{j=1}^{n}\max\left(0, rt_i \cdot \frac{(v_i - v_j)}{v_j}\right) \\
pdc_i &= -\frac{1}{n}\sum_{j=1}^{n}\min\left(0, rt_i \cdot \frac{(v_i - v_j)}{v_j}\right) \\
MPC &= \frac{1}{n}\sum_{i=1}^{n}(psc_i + pdc_i)
\end{aligned}
\tag{3}
$$

SSHR & SAHR

Two measures are proposed in [21], sum of shortest headway reciprocals (SSHR) and sum of arrival headway reciprocals (SAHR). SSHR reacts to both difference in speed between trains and the spread of the departures/arrivals over time. The SAHR considers only headways at arrival under the assumption that headway at arrival is more important than at departure when secondary delays are created.

The measures are calculated for each line section separately. The reason why the measure is calculated for each line section separately is its headway measuring feature, which means it can differ from one line section to another. The SSHR is always equal to or larger than the SAHR. The difference between the two may be large if the timetable consists of trains with large speed differences. SSHR is the same as SAHR when all trains travel at the same speed. As defined here, they are not independent of traffic density or number of trains (headways) included in the calculation. In order to avoid the dependency of the number of trains, always 6 trains are used in this paper in the calculation of the SAHR and SSHR, no matter if the number of trains in each cycle is 1, 2 or 3. The possibility to measure uneven headways can give an advantage compared to the other measures that does only measure heterogeneity in speed.

$$SAHR = \sum_{i=1}^{n} \frac{1}{h_i^A} \qquad h_i^A : \text{ headway at arrival between trains } i \text{ and } i+1 \text{ on the track section}$$

$$SSHR = \sum_{i=1}^{n} \frac{1}{h_i^-} \qquad h_i^- : \text{ smallest scheduled headway between trains } i \text{ and } i+1 \text{ on the track section} \tag{4}$$

4 Simulation Results

Traffic Density

The bar graph below summarizes the results of one of the timetables consisting of freight trains, IC trains and high speed trains, i.e. one of the more heterogeneous timetables. The graph shows clearly how both the timetable and the trains in operation are affected when traffic density is increased. The figures are mean values for all train types combined. The bars showing the available allowance include scheduled delay, hence the dramatic increase in available allowance at stations as traffic density grows and overtakes become more frequent. The secondary delays at stations increase somewhat for every increment in traffic density while the secondary delays on line sections increase slowly at first and then more dramatically at the highest two levels. Secondary delays at stations are mainly caused by low priority trains waiting to be overtaken by high priority trains, while on line sections, trains tend to interfere with other trains more freely, regardless of priority.

It is also evident in the figure that the allowance at stations that is used to reduce delay increases with higher traffic densities, while the used running time allowance remains approximately constant. The main reason for this is the increase in available allowance at stations. For the first four timetables, the increase in used allowance manages to compensate for the increase in secondary delay, and it is not until the final two timetables that the exit delay starts to increase. All in all, the graph shows how allowance and delays interact and the result thereof, i.e. exit delay.

One methodological aspect apparent from figure 2 is that all types of primary delays are as good as constant for all simulations. This is intended and shows that enough replications have been simulated to achieve stable mean values. Another is that the bars showing the delays and used allowance sums up to the same value, which shows that the definitions of secondary delays are consistent with the definitions of used allowance.

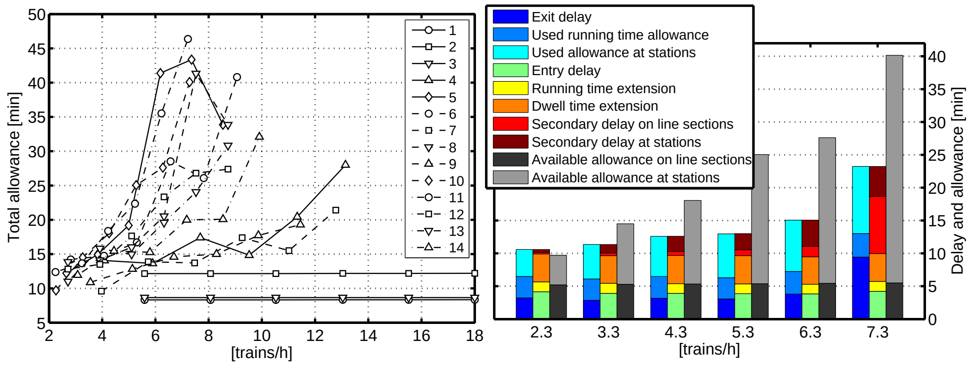

Figure 2: Left, total allowance for all timetable types as function of traffic density. Right, simulation results for timetable type 10 (high-speed, intercity, freight trains), mean values for all train types.

Secondary delays and heterogeneity is the focus of this paper. Figure 3 below shows secondary delays as function of traffic density for all 14 types of timetables. Left figure shows delays on line sections and the right at stations. Several things are worth commenting on:

Secondary delays on line sections seem to increase exponentially with traffic density, while at stations the increase shows a more linear behaviour. The probable explanation is that the increase in allowance at stations prevents a rapid growth of secondary delays, while on line sections no such extra allowance exists. Secondary delays at stations are closely correlated to the scheduled delay, figure 2. More scheduled delay is in this case the same as more allowance at stations, which have the main effect of reducing secondary delays at stations. This is the explanation why in some cases, secondary delays can decrease locally as traffic density increases. However, in some extreme cases there is also a negative effect of slower trains being scheduled to wait for long times, can be seen as large line delays of timetable 6. The reason is that sidetracks will be occupied for a large percentage of the time and thus unavailable for unplanned overtakes. This is further discussed in [10].

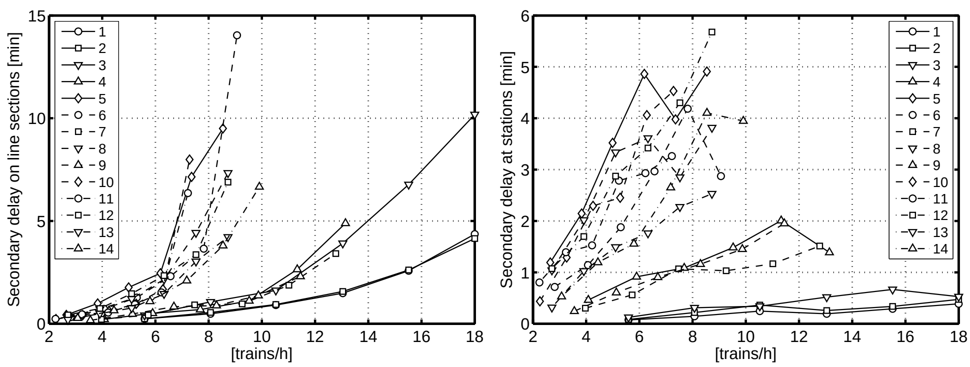

Figure 3: Secondary delays on line sections (left) and at stations (right).

It is possible to distinguish three separate groups in the figures. The first with the lowest delays are the completely homogenous timetables (1-3). The second group consists of timetable (4, 7, 9) and corresponds to timetables with only passenger trains. In the final group, all timetables that are a mix of freight trains and passenger trains (5, 6, 8, 10-14).

Of the completely homogenous timetables, freight trains seem to suffer from more delays than the passenger trains. At stations this can be explained by that the passenger trains have more scheduled stops and that the scheduler starts to schedule trains on both tracks alternately, this can be seen as a slight decrease between 11 and 13 trains/h for timetable 1 and 2. On line sections the difference is more significant and is probably due to a combination of smaller possibilities to recover from delays at stops, poor acceleration performance and longer time to clear the signalling block sections due to lower top speed and longer trains. This behaviour is an example where Vromans [20] measure for headway buffers can be used to add valuable information to the analysis.

Train Types

Several aspects have to be considered when capacity is defined and conditions have to be applied to both the timetable and to the train operation. For timetables, these conditions are derived from demand and may for example include clock-face timetables, and scheduled delay. Level of acceptance of operational delays might be the most important condition on the train operation. Both the scheduled and operational delay goes up when traffic density is increased. In this case the slower train types have lower priority both in the scheduling procedure and in the simulation. The consequence is that slower trains receive both scheduled delay and operational delay, while faster trains suffer only from operational delay. This indicates that the results have to be separated according to train type in the analysis.

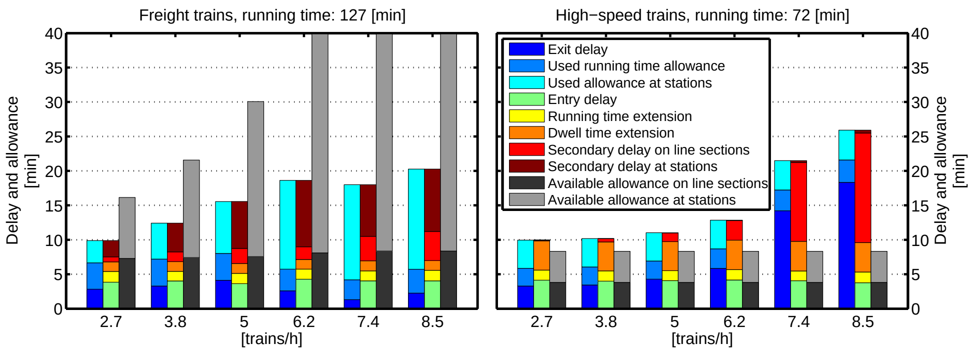

Figure 4: Results separated for individual train types in timetable type 5. Values for the bars ending outside the left figure are (left to right) 75, 82 and 62 minutes.

Figure 4 shows results from timetable type 5, separated for each train type. It consists of freight trains and high-speed trains and is one of the most heterogeneous timetables. The left diagram shows results for freight trains and the right for high speed trains. Several interesting observations are worth commenting:

As mentioned before, it is only the freight trains that receive scheduled delay due to more frequent overtakes at higher traffic densities. The scheduled delay is substantial and becomes as much as 82 min, outside the figure, which corresponds to an increase in scheduled running time of 65%. The scheduled delay is closely connected to the scheduling scheme. It is possible that if small scheduled delays are accepted also for high priority trains, scheduled delay for low priority trains would decrease significantly.

Worth commenting is also the fact that the scheduled delay for freight trains decrease when traffic density increase from 7.4 to 8.8 trains/h. This is an effect of using cyclic timetables and an infrastructure with the ovartakings stations spaced equidistantly. A shorter headway may cause a better timing at overtakes, i.e. the freight trains have to wait for a shorter time before the high-speed trains arrive, while the number of overtakes required remains the same.

The large difference in used allowance between the train types is explained by the difference in available allowance. For freight trains the increase in used allowance at stations is more than 11 minutes, which more than well covers for the increase in secondary delays. For high speed trains there is almost no increase in used allowance at stations, and a very small increase of used running time allowance. The limited increase is explained by that most of the allowance is already used, even at low traffic densities.

Freight trains receive most of their secondary delay at stations, but some on line sections as well. High-speed trains get it only on line sections. The reason for this is that lower priority trains have to wait at stations to be overtaken by faster high priority trains. The efficient dispatching and the fact that dwell time extensions rather than departure delays have been used in the simulation, has the effect that the high priority trains get next to no secondary delays at stations. Looking at the secondary delays in total, freight trains get some delays even at low traffic densities. It increases with traffic density, but seems to level out somewhat. For high speed trains, the secondary delays are at first almost non-existent, but at around 5 trains/h the secondary line delays start to increase quite fast. The rapid increase is probably explained by the limited capacity of the two track stations that only allow one train to be overtaking at a time. At 7.4 and 8.5 trains/h, overtakes are scheduled at every station.

The exit delay does also differ between the two train types and is explained by the difference in used allowance. The development of the exit delay of the high-speed trains follow quite well that of the secondary delays, which is natural since nothing else changes much. The freight trains however, manage to keep the exit delay constant, or even reduce it slightly, as the traffic density increases. Even though figure 4 does not show the same timetable as figure 2, the behaviour of the involved train types are the same. In figure 2, the exit delays remain stable at first, and then start to increase in union with the secondary delay on line sections. Looking at figure 4, this behaviour can now be explained by that it is the secondary line delays of the high priority trains that starts to go up, and since they cannot use any allowance for recovery, so does the exit delay.

5 Regression Analysis of Heterogeneity Measures

Stepwise multilinear regression is used to find out how the calculated parameters correlate to secondary delays. The algorithm uses p-values for the F-statistic to decide which parameters to include in the model [1]. The default settings are used, i.e. an initial model with no terms, an entrance tolerance for the p-value of 0.05 and an exit tolerance of 0.10. For the exponential models, the natural logarithm is applied to the delays before the regression. Note that the Root Mean Square Error (RMSE) values presented for the exponential models are affected accordingly. To make interpretation of the results easier, the regression equation is kept simple and is limited to include first order terms and their interaction effect. Even if including higher order terms probably would give a better fit, the aim is not to create a model explaining delays, but rather analyse the explanatory power of the different heterogeneity measures.

Measures Independent of Traffic Density (SL, SR, MDFR, MPC)

Of the five measures previously discussed, four are independent of traffic density. It is therefore possible to compare their values for the 14 different types of timetables. Figure 5 below shows calculated heterogeneity for SL, SR, MDFR and MPC, normalised to range from 0 to 1.

All measures assign lowest value to the same timetables, all timetables consisting of only one train type (1-3). SL differs from the other measures by not explicitly using the speeds in the calculation. It assigns the highest values to timetable 10 and 11, the only ones containing three train types. All other timetables except the completely homogenous have two train types and are considered equal by SL. This makes it that all 14 timetables are represented by just three unique values.

For the other measures, timetable type 5 with as many high-speed trains as freight trains, has highest heterogeneity. SR also assigns the highest value to timetable 8 and 10-12, which all contain high-speed trains and freight trains. SR has four different levels, corresponding to the three combinations of the three different train types and the case with only one train type (ratio: 1). MDFR has six levels and MPC seven. Ranking the timetables using the different measures gives that timetable 4, 5 and 6 are valued differently. Timetable 4 is considered by SR to have the same heterogeneity as 7 and 9, while both MDFR and MPC considers timetable 4 to be more heterogeneous than 7 and 9. The same is true for timetable 6 compared to 13 and 14. MDFR puts the same value on timetable 6, 8 and 10-12 while MPC considers 6 so be more homogenous than the others. Finally, timetable 5 is the single most heterogeneous timetable according to MDFR and MPC, while SR yields the same value for 5 as for 8 and 10-12. Despite this, it is possible to rank the timetables from homogenous to heterogeneous and preserve the individual ranking of all measures (not including SL).

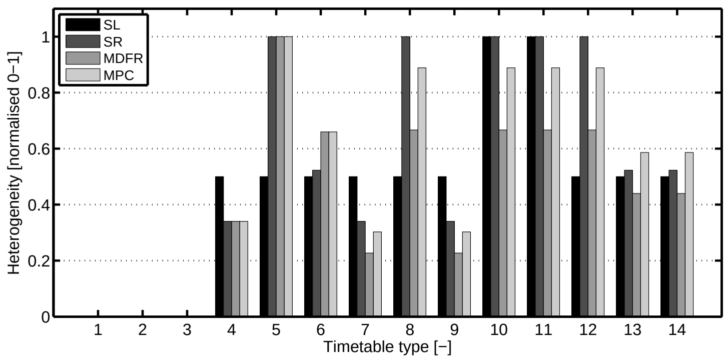

Figure 5: Normalised heterogeneity values for different timetable types. True range is: SL: 1-3 [-], SR: 1-1.77 [-], MDFR: 0-55 [min], MPC: 0-0.46 [h].

An interesting comparison can be made between 5 and 10 (or 11). Timetable 5 consists of high-speed and freight trains and 10 of high-speed, intercity and freight trains. When intercity trains are added (lower speed than high-speed and higher than freight trains), heterogeneity decreases according to MDFR and MPC, but not for SR. The same is true if the added train is of a train type that does already exist in the timetable, e.g. timetable 4 and 7 and 8. Another observation is that none of the measures distinguish between timetables 7 and 9, 8 and 12 or 13 and 14 respectively, i.e. it does not matter if it has two trains of one type and one of another, or the other way around.

In general, the SR, MDFR and MPC have similar behaviour. Having more levels might suggest a higher resolution and a better ability to capture relevant information. SR only looks at two trains while the other measures take all trains into consideration. MDFR and MPC always give the same result for timetables with two train types or less, where the only difference is a factor of 120 (2 hours).

Table 4: Regression results for SL, SR, MDFR and MPC. z : secondary delay, x : traffic density, y : heterogeneity.

20 km				z = a+bx+cy+dxy					
Location	Model	Primary delay	Measure	a	b	c	d	r^2	rmse
Line	Linear	Low	SL	-0,52	0,17	-	-	0,43	0,76
			SR	-1,34	0,22	0,82	-	0,49	0,72
			MDFR	-1,55	0,23	1,32	-	0,53	0,69
			MPC	-1,55	0,23	1,07	-	0,52	0,70
		High	SL	-1,81	0,38	-	0,45	0,37	2,12
			SR	-2,93	0,44	-	0,62	0,55	1,79
			MDFR	-3,12	0,44	-	0,85	0,62	1,63
			MPC	-3,21	0,44	-	0,71	0,60	1,68
	Exponential	Low	SL	-2,91	0,23	-	0,13	0,63	0,67
			SR	-3,36	0,25	-	0,20	0,77	0,53
			MDFR	-3,40	0,25	-	0,27	0,81	0,49
			MPC	-3,45	0,25	-	0,23	0,80	0,49
		High	SL	-1,97	0,20	-	0,24	0,63	0,68
			SR	-2,45	0,23	-	0,30	0,85	0,43
			MDFR	-2,46	0,23	-	0,39	0,89	0,38
			MPC	-2,55	0,24	-	0,34	0,90	0,36
Station	Linear	Low	SL	0,21	-	-	0,17	0,36	0,46
			SR	-0,20	0,03	-	0,22	0,80	0,26
			MDFR	-0,15	0,02	-	0,27	0,77	0,28
			MPC	-0,23	0,03	-	0,24	0,83	0,24
		High	SL	0,47	-	-	0,45	0,41	1,09
			SR	-0,29	0,04	-	0,55	0,84	0,56
			MDFR	0,06	-	-	0,66	0,83	0,58
			MPC	-0,40	0,04	-	0,60	0,88	0,49
	Exponential	Low	SL	-1,87	-	-	0,38	0,48	0,79
			SR	-3,18	0,12	1,26	0,26	0,77	0,53
			MDFR	-3,24	0,12	1,95	0,27	0,77	0,54
			MPC	-3,57	0,14	1,86	0,24	0,83	0,46
		High	SL	-0,98	-	-	0,39	0,56	0,70
			SR	-1,12	-	-	0,39	0,80	0,47
			MDFR	-1,09	-	-	0,48	0,79	0,48
			MPC	-1,19	-	-	0,43	0,84	0,42

Table 4 summarizes the results from the regression for the four heterogeneity measures. It is based on all simulations of the infrastructure variant with 20 km inter-station distance. Output from the simulation is separated into secondary delays on line sections and at stations. The delays are mean values and at this stage delays are not separated according to train type. Results for the 40 km infra is not showed, but are similar with respect to R^2 values.

Looking at the R^2 values, the exponential models seem to be more appropriate than the linear for secondary delays on line sections. At stations, the difference is not as large, but the linear models perform slightly better. The same is observed in all other regressions performed in this paper. Hence from now on only exponential models for line delays and linear models for delays at stations will be discussed.

The fit of the models are in general better when higher primary delays are applied. A possible explanation is that when delays are higher, the timetable has less impact on the behaviour of the trains when they are so late that they are not close to their scheduled train slot. The effect of different train speeds however, is still present.

In many models coefficient *c* is not significant. Coefficient *c* corresponds to the pure effect of the heterogeneity measure. Heterogeneity is still present in the model via the interaction term, coefficient *d*.

MDFR and MPC: the interaction term *d* grows with higher primary delays, both on line sections and at stations. At the same time term *b*, traffic density, remains constant or even decrease somewhat. The combination of traffic density and heterogeneity seems more important than traffic density alone when primary delays are high.

MDFR and MPC: coefficient *b* is larger relative to *d* on line sections than at stations. Apparently, secondary delays on line sections are not as closely dependent on heterogeneity as secondary delays at stations.

On line sections, MDFR and MPC perform equally well and better than SR. At stations, MPC is the best, followed by SR as second best and MDFR as the third best measure. SL, the number of speed levels, has the worst performance in all cases. A residual analysis shows how the models using SR and MDFR fit the different timetable types at different traffic densities, figure 6.

Secondary delays on line sections: Both models underestimates delays in timetable 3, consisting of only freight trains. It is natural, since none of the parameters in the regression, traffic density and heterogeneity, covers effects on delays due to train type characteristics, as previously discussed. Both models overestimate delays in timetable 4, MDFR performs slightly worse. MDFR has a lower mean value of all timetables than SR, which explains why MDFR overestimates more that SR, despite both measures giving the same score to timetable 4. The same principle explains the difference in timetable 5.

Timetable 6 is the first where the two measures give different scores. The lower score given by SR makes it underestimate the delays, while MDFR has a higher score and smaller residuals. The delays at the highest traffic density are considered to be an outlier (red) by SR and closely so by MDFR. Looking at figure 6, the last increase in traffic density leads to a very dramatic increase in line delays, hence the outlier. The same behaviour is seen in timetable 10 and 11. Many timetables are extreme at the highest traffic density, in the sense that overtakes, sometimes double overtakes, are scheduled at almost every station. This leads to that capacity at stations becomes a problem and that faster trains get trapped behind slower trains for several line sections before being allowed to pass.

SR overestimates delays in timetable 10 and 11, the timetables containing all three train types. It is interesting to see that MDFR performs better due to that it assigns a lower heterogeneity to timetable 10 and 11 than to timetable 5.

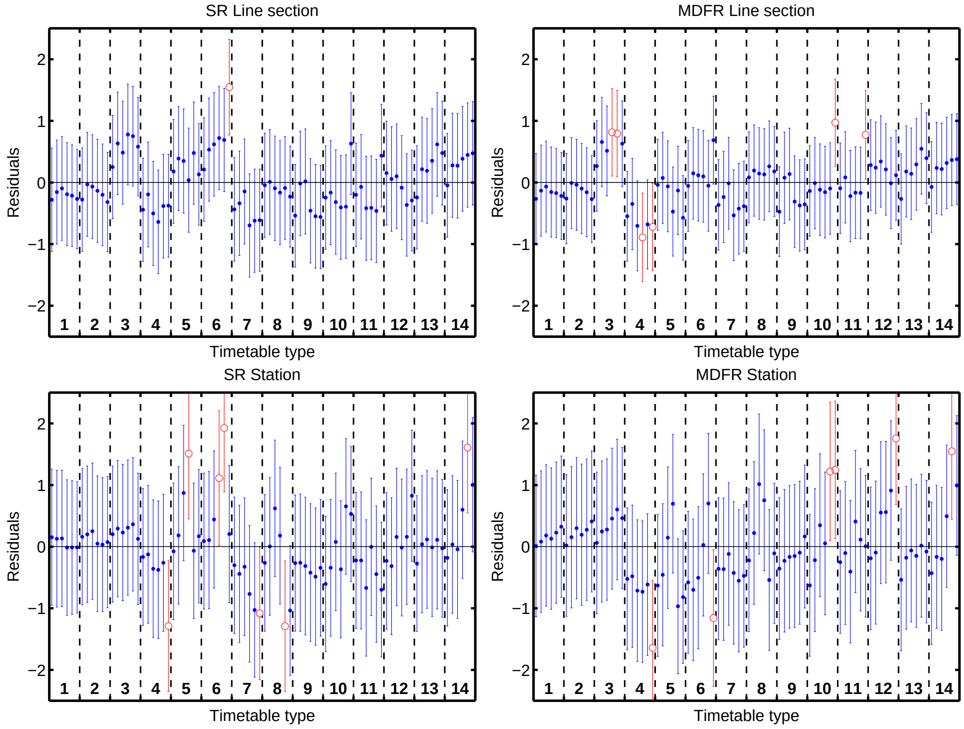

Figure 6: Residuals [min] for models using SR and MDFR. Line sections (top) and stations (bottom). High primary delay level. Negative residuals indicates that the model overstimates delays. Outliers are indicated in red color (95%). For each timtable type, traffic density increase in the rightward direction.

Looking at the residuals at stations, the main conclusion is that variation in available allowance, i.e. the number of scheduled overtakes and how long trains are scheduled to wait at stations, have a large impact on secondary delays. Most of the outliers are connected to extreme increase, or sometimes decrease, in allowance when traffic density increase. In general, if traffic density is increased without a corresponding increase in allowance at stations, more secondary delays will occur. The opposite happens if allowance increase dramatically from one step in traffic density to the next. To make things more complicated, it is also of relevance if the allowance is due to a few badly timed overtakes or many with better timing. This may be one of the reasons why using free running times and speeds, rather than scheduled, when the heterogeneity measures are calculated produce better results.

SSHR and SAHR

An advantage of SSHR and SAHR is that they also measure heterogeneity in headway. To the other measures that only look at speed differences, all line sections look the same. But as headways may differ from line section to line section, SSHR and SAHR can be used to predict how much delays will occur on each individual line section separately. For this reason, SSHR, SAHR and secondary delays for each line section and station is used in a first regression analysis. The fit turned out to be quite poor, around 0.6 R^2 or less.

However, the main reason is that delays are not stochastically stable on the level of individual line-sections. The experiment was designed to give stable results taking the total delay of all line sections or stations. The solution is to make more replications, but this was not done in this paper. Instead, the same delay inputs as in the other regression models are used, i.e. the total secondary delay on all line sections and stations. The SSHR and SAHR are calculated as the mean of all line sections. Unfortunately, this reduces the variance of the measures and the possibility to determine their true power.

Table 5: Regression results for SAHR, SSHR. *z* : secondary delay, *x* : SAHR, *y* : SSHR.

				z = a + bx + cy				
Location	Model	Primary delay	Infra	a	b	c	r^2	rmse
Line	Exponential	Low	20 km	-3,31	-	0,99	0,82	0,46
			40 km	-4,03	-	1,17	0,81	0,58
		High	20 km	-2,22	-0,34	1,31	0,90	0,36
			40 km	-2,66	-	1,08	0,88	0,40
Station	Linear	Low	20 km	0,17	-0,74	0,90	0,68	0,33
			40 km	-0,02	-0,46	0,59	0,79	0,19
		High	20 km	0,62	-2,04	2,34	0,77	0,68
			40 km	0,20	-1,19	1,39	0,86	0,35

The results show a pretty good fit in most cases. It may be a bit surprising that it correlates well with delays at stations, but uneven headways may also increase the risk for overtakes, hence delays at stations. On line sections, SSHR dominates and SAHR and SSHR are both present at stations. The negative coefficient of the SAHR is due to that it correlates to SSHR. Highest delays are achieved when SAHR is small and SSHR large, which is the case when large speed differences are present, compare with the measure for homogeneity proposed by Landex [7]; the quotient of SAHR and SSHR.

Inter-Station Distance and Number of Overtakes

To establish the effect of increasing inter-station distances, it is necessary to separate the analysis with respect to train types, or rather for faster and slower trains. Figure 7 illustrates an example of results for one timetable type. It shows that faster high priority trains suffer from increased delay on line sections, while slower low priority trains in some cases benefits some, both on line sections and at stations.

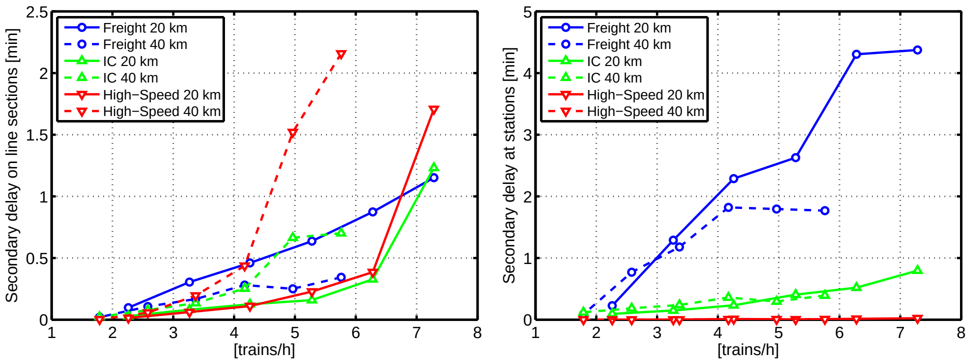

Figure 7: Results for timetable type 10, separated with respect to train type and infrastructure variant (20 km: solid / 40 km: dashed). Low primary delays.

A regression analysis is performed in the same way as before, but delays are calculated for each train type separately. Beside mean delays, the *psc* and *pdc* for each train type is used, rather than MPC that is the mean of all train types. This way the same regression model can be used to explain the behaviour of the different train types. For example, IC trains may act as the faster train in a timetable with IC and freight trains, but as the slower in one where high-speed trains are present. This is then represented by different values of the *psc* and *pdc*.

The regression model consists of three factors, traffic density, number of times the train type is passed by other trains as well as the number of times it is scheduled to pass other trains. As before, an exponential model is best for line sections and a linear for stations. The model for the line has worse fit than the one for the station, table 6. Two possible reasons for this is that all train types suffers from line delays, while it is almost only the low priority train types that suffers from delays at stations. The other is that the line delays have different shapes depending on train type. It is rather exponential for the faster high priority trains, while it is more linear for the slower. The reason is probably that the delays have different sources. For trains with higher speed, the line delay comes from other trains of the same speed and being stuck behind slower trains, while the primary source for slower trains are other slow trains interfering and being forced to decelerate and accelerate due to unplanned overtakes. Line delays for slower trains will also occur at extreme traffic densities when they sometimes have to wait for a side-track to clear in order to be overtaken.

Table 6: Regression results when train types are characterised by the number of times they pass, and are passed by other trains. *z* : secondary delay, *x* : traffic density, *xy1* : number of times passing other trains, *xy2* : number of times passed by other trains.

				z = a+bx+cxy1+dxy2					
Location	Model	Primary delay	Infra	a	b	c	d	r^2	rmse
Line	Exponential	Low	20 km	-3,72	0,27	0,42	0,52	0,69	0,70
			40 km	-4,31	0,30	1,02	0,46	0,62	0,91
		High	20 km	-2,61	0,25	0,61	0,64	0,75	0,62
			40 km	-2,73	0,24	1,03	0,51	0,75	0,59
Station	Linear	Low	20 km	-0,14	0,02	-	0,94	0,87	0,42
			40 km	-0,34	0,03	0,11	1,02	0,90	0,30
		High	20 km	0,00	-	-	2,37	0,91	0,85
			40 km	-0,50	0,04	0,17	2,40	0,94	0,51

Looking at the coefficients in the regression models, it can be hypothesized that coefficient *b* correlates to the interaction with other trains of the same speed (in a homogenous timetable *y1* and *y2* are zero). Coefficient *c*, faster trains being obstructed by slower trains. Coefficient *d*, slower trains being forced to decelerate and accelerate due to unscheduled overtakes. However, since the models do not fit data very well, 0.62-0.75 R^2, correct interpretations may be hard. At stations, the model is dominated by coefficient *d*, i.e. the number of times you are expected to be passed by faster trains.

The effect of primary delays is on line sections mainly seen as an increase in the intercept, *a*, but also possibly in *c* and *d*. In this case it is a limitation that the types of primary delays are not varied individually. At stations however, increased primary delay makes *d* grow, which is not surprising since it is still mostly slow low priority trains that gets delayed at stations, regardless of the level of the primary delay.

Inters-station distance does not affect delays at stations much. On line-sections however, the major difference is that coefficient *c* increase when inter-station distance increase, which seem natural if it represents faster trains being delayed by slower. Coefficient *d* decrease slightly, and together with a lower intercept, line delays seem to drop some for low speed trains at the 40-km infrastructure, which can also be seen in figure 7. Coefficient *b*, interaction between trains with the same speed, remains constant.

6 Conclusions

The five measures of heterogeneity analysed in this paper are all able to explain secondary delays from the simulation to varying degrees. Four of the measures only look at differences in speed: number of speed levels (SL), the speed ratio of fastest to slowest train (SR), mean difference in free running time (MDFR) and mean pass coefficient (MPC) that correlates to the number of overtakes. Of these four, SL shows significantly lower ability to explain secondary delays than the rest. This is probably explained by the fact that it is the only measure that does not take the actual speeds of the trains into account, just the number of different speed levels. Of the other measures, MPC performs best, closely followed by MDFR and SR.

Sum of shortest headway reciprocals (SSHR) and sum of arrival headway reciprocals (SAHR) also show good performance. However, due to limitations in the experimental setup, further analysis is required to determine the potential of their capability to measure heterogeneity in terms of unevenly distributed headways.

MPC is a heterogeneity measure developed in this paper that is designed to correlate to the number of overtakes that is required in a cyclic timetable. It is efficient in explaining the total amount of secondary delays on line sections and at stations, figure 8. An advantage is that it can be calculated for all train types in the timetable individually, which is valuable since delays affect slower and faster differently. This is exploited in an analysis where delays are separated for different train types and their train slots characterised by the number of times they are expected pass other trains as well as the number of times they are expected to be passed by other trains. Both faster and slower trains suffer from delays on line sections, although delays are in general higher for the faster trains. At stations however, almost only slower trains suffer from secondary delays.

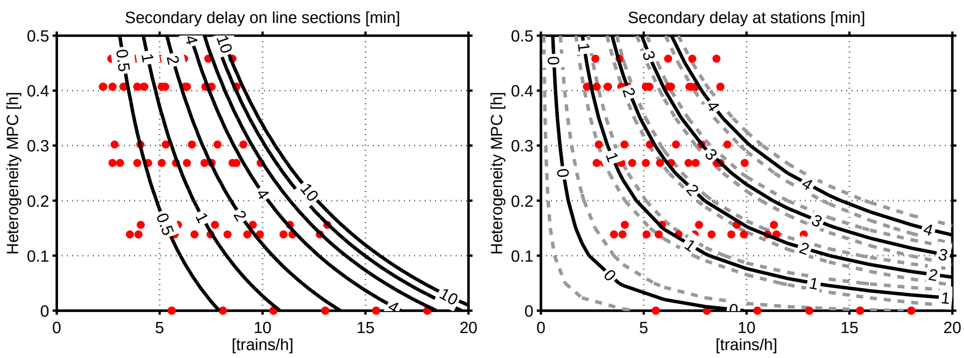

Figure 8: Contours of delay models using MPC as heterogeneity measure (black lines). Simulated values (red dots). Grey contours indicate the 95% prediction intervals.

Inter–station distance does not have any significant effect on the performance of the heterogeneity measurers. When train types are separated in the analysis, the most significant effect of increasing the inter-station distance is that faster trains suffer from more delays on line sections while the effect on slower trains are not as clear, if possible they even seem to benefit some.

Other conclusions are that all measures perform better when higher primary delays are applied in the simulation model, which is probably due to that the scheduled timetable affects secondary delays less when more trains are late. On line sections secondary delays show an exponential growth; this is in line with previous research. At stations however, a linear model seems more adequate. The difference is that increasing scheduled delay at stations acts as allowance and helps to reduce secondary delays at stations when traffic density grows.

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